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MORPHISMS AND PROPERTIES OF NON-ASSOCIATIVE ALGEBRAIC SYSTEMS WITH MOUFANG TYPE CONDITIONS

111.03 MATHEMATICAL LOGIC, ALGEBRA AND NUMBER THEORY

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CONTENTS

1. KEYWORDS

keywords: quasigroup, loop, group, groupoid, isotop, automorphism, left identity element, right identity element, pseudo-automorphism, left Bol quasigroup (right), Moufang quasigroup, WA-quasigroup, CI-quasigroup, i-quasigroup, medial quasigroup, Neumann quasigroup, left-transitive quasigroups, G-property.

2. RESEARCH GOALS AND OBJECTIVES

The topicality and importance of the problem addressed. Quasigroup theory began in the 20-30s of the XX century, when after the fundamental works of David Hilbert in the late nineteenth century on the axiomatization of mathematics and, in particular, on the axiomatization of geometry, there have appeared works related to different axiom systems, in the main to the axiom systems of various geometries, including Euclidean geometry, projective geometry, Lobachevsky geometry, mostly, in dimensions 2 and 3.

The very term "quasigroup" appeared in Ruth Moufang's paper [1] on the coordinatization of projective planes. We can also say that the term “quasigroup” appeared when studying the question of the independence of axioms in the axiom systems of the projective plane.

In her works Moufang understood by quasigroup the object, which is now called a Moufang loop. In modern terms, she defined the Moufang loop as an IP-loop $(Q,\cdot)$ with some “weak associativity” identity.

The next most important paper on the subject of quasigroups appeared two years after Moufang′s: “Gewebe und Gruppen” by Gerrit Bol (1937). Bol′s approach is from a web-geometrical point of view. He constructs three new configurations U1, U2, U3 and asks whether the closure of these three figures implies associativity. He answers that question in the negative, and shows that the three U figures together imply only the law $a\,[b\,(cb)] = [(ab)\,c]\,b$, which is precisely one of the Moufang identities. To demonstrate this fact, Bol given an example constructed by Zassenhaus. This example (of order 81) was, in fact, the first example of a non-associative commutative Moufang loop. Further, Bol explains the algebraic meaning of of each of the U figures and shows that U1 and U2 correspond to laws that we now call the right Bol and the left Bol identities, respectively: $a\,((bc)\,b) = ((ab)\,c)\,b$ și $(b\,(cb))\,a = b\,(c\,(ba))$. It was Zassenhaus, again, who soon constructed the first example of a right Bol loop.

Quasigroups have a variety of applications in statistics (experiment planning theory) [2], the theory of differential equations, differential geometry [3], hyperbolic geometry [4], physics [5], code theory [6], cryptography [7].

For quasigroups, especially considering their relations to combinatorics, various morphisms have been determined and actively researched, among which we mention isomorphisms, automorphisms, isotopes, autotopies, isostrophies, autostrophies, generalized isotopes. Automorphisms and groups of loop automorphisms were investigated by A. A. Albert already in the first works in quasigroup theory [8].

Almost all well-known (classical) classes of quasigroups and loops possess the property of invertibility. Most often these quasigroups possess one of the 4 best known properties of invertibility, namely $IP-, LIP-, RIP-, WIP-$ or $CI-$ inverse property.

IP-loops and LIP-loops were defined in the work of R. Moufang [1]. WIP-loops were defined and researched in the work of R. Baer [9]. CI-loops are classical objects of quasigroup theory. This class of loops was defined by Rafael Artzy [10]. V. D. Belousov and B.V. Tsurkan defined CI -quasigroups in [11].

The important problem is to investigate the properties of non-associative algebraic systems with Bol-Moufang type identities and to find the relations between these subclasses and the main classes Moufang and Bol, applying the morphisms.

The purpose and objectives of the thesis. The aim of the paper is to investigate the morphisms and properties of non-associative algebraic systems with Moufang-type identities. To achieve this goal, the following objectives have been defined:

(1) research on the relations of WA-, CI-quasigroups, transitive on the left and Neumann with the quasigroups Moufang, Bol on the left, on the right, etc.;
(2) research of quasigroups with any of the 60 classical Bol-Moufang identities listed in [12] at the existence of the unit;
(3) research of morphisms, properties, relationships with other classes of quasigroups of newly defined quasigroups (i-quasigroups and $OWIP$-quasigroups);
(4) research on the G-properties of left transitive quasigroups and Neumann quasigroup.

3. SCIENTIFIC RESEARCH METHODOLOGY

Constructions and demonstration methods are based on the application of the notions of quasigroup, parastrophe, local units, isotopes, autotopes, nuclei, pseudo-atomorphisms, G-properties.

Scientific novelty and originality consist in obtaining new theoretical results with applications in other fields. All the results presented in the thesis are new and original. Different classes of quasigroups (WA-, CI-, left transitive quasigroups, Neumann quasigroups) were researched. Two new classes of quasigroups ($OWIP$-quasigroups, i-quasigroups) were introduced and researched. Classes of quasigroup isotopic to a group were investigated. The properties of some classes of invertible quasigroups are described. The connections of the quasigroup classes investigated with the classical quasigroups Moufang, Bol and other quasigroup classes were researched. The forms of automorphisms, pseudo-automorphisms and quasiautomorphisms of these quasigroups were determined.

The important scientific problem solved is the application of morphism type relations to the research of the properties and units of non-associative algebraic systems with Moufang type conditions that lead to the description of new important relations between quasigroup classes.

Theoretical importance and applicative value of the paper it is determined by obtaining new results in the research of non-associative systems of Bol-Moufang type. The paper theoretical character. The methods developed in the paper allowed solving the problems.

Approval of results. The scientific results obtained were presented and approved during the Special Session of the Seminar "Algebra and Mathematical Logic", dedicated to the memory of Professor V. Belousov, the Vladimir Andrunachievici Institute of Mathematics and Computer Science of the Academy of Sciences of Moldova. The main results included in the thesis were presented at the following scientific conferences:

- The 25th Conference on Applied and Industrial Mathematics CAIM 2017, September 14–17, 2017, Iași.
- International conference on mathematics, informathics and information technologies dedicated to the illustrious scientist Valentin Belousov, April 19-21, 2018 Bălţi.
- The 26th Conference on Applied and Industrial Mathematics CAIM 2018, September 20-23, 2018, Chişinău.
- International conference "Mathematics&Information technologies: research and education", MITRE-2019, Moldova State University, June 24, 2019, Chișinău.
- LOOPS 2019 Conference, Budapest University of Technology and Economics, July 7-July 11, 2019, Hungary.
- The 5th International Conference of Mathematical Society of the Republic of Moldova, dedicated to the 55th anniversary of the foundation of Vladimir Andrunachievici Institute of Mathematics and Computer Science (IMCS-55), September 30, 2019.
- "Tendințe contemporane ale dezvoltării științei: viziuni ale tinerilor cercetători": Scientific Conference of PhD Students (with international participation), 6th edition, Chisinau, June 15, 2017.

- The Fourth Conference of Mathematical Society of the Republic of Moldova: dedicated to the centenary of Vladimir Andrunachievici (1917-1997): Proceedings CMSM 4, June 28 – July 2, 2017, Chișinău.
- “Tendinţe contemporane ale dezvoltării științei: viziuni ale tinerilor cercetători”: Scientific Conference of PhD Students (with international participation), 6th edition, Chisinau, June 15, 2018.

Thesis publications. A total of 16 scientific papers have been published, comprising 6 articles in peer-reviewed journals [13], [14], [15], [16], [17], [18](2 articles without co-authors) and 10 abstracts at scientific conferences [19], [20], [21], [22], [23], [24], [25], [26], [27], [28] (7 abstracts without co-authors).

Thesis structure and thesis volume. The thesis is written in Romanian and contains: introduction, four chapters, general conclusions and recommendations, 98 bibliographic titles, 95 pages (including 86 pages of basic text).

4. SYNTHESIS OF CHAPTERS

The structure of the thesis is represented by four chapters, which contain the theoretical results obtained in the research of the properties of non-associative algebraic systems with Moufang type identities.

The introduction formulates the topicality and importance of the research topic. In addition, the objectives, scientific novelty and originality are mentioned. The scientific problem studied is presented with emphasis on the importance of the theoretical and applied value of the paper. A short analysis of the issues and publications on the topic of the thesis is presented. This section concludes with a summary of the content of the paper.

The first chapter – Analysis of the situation in the field of non-associative algebraic systems with Moufang type conditions consisting of six paragraphs, has an introductory character. In this chapter analysis of the fundamental algebraic concepts is made. Known results are presented which are important in the following chapters.

In the **second Chapter - About some classes of quasigroups with invertibility properties ($\boldsymbol{WA-}$, $\boldsymbol{OWIP-}$, $\boldsymbol{CI-}$ quasigroups)** – two known classes of quasigroups which have an inverse property are investigated; a new class of quasigroups is introduced, a generalization of WIP-quasigroups. The chapter consists of seven paragraphs in which objectives 1 and 3 are achieved. The results of this chapter have been published in [19], [23], [22], [14], [15].

The first three sections are dedicated to the results obtained in the WA-quasigroup research.

Quasigroup $(Q,\cdot)$ with identities $xx \cdot yz = xy \cdot xz$ and $xy \cdot zz = xz \cdot yz$ is called a WA-*quasigroup* or *semimedial quasigroup* (in short: SM-quasigroup) (see [29]).

Applying information from [30] that *any loop, isotopic of WA-quasigroup, is a Moufang commutative loop*, the following facts have been demonstrated: Lemma 2.1.2., Lemma 2.1.3., Lemma 2.1.4..

Lemma 2.1.2. Any WA-quasigroup with a left identity element is a left Bol quasigroup.

Lemma 2.1.3. Any WA-quasigroup with a right identity element is a right Bol quasigroup.

Lemma 2.1.4. Any WA-quasigroup $(Q,\cdot)$ with the left (right) inverse property is a left (right) Bol quasigroup.

From Lemma 2.1.4. and Lemma 1.5.1., Corollary 2.1.2. results.

Corollary 2.1.2. Any WA-quasigroup, which is an IP-quasigroup, is a Moufang quasigroup.

By researching the derivative operations for the WA-quasigroup $(Q,\cdot)$, we obtained Theorem 2.1.2.

Theorem 2.1.2. Let $(Q,\cdot)$ be a WA-quasigroup. Then:

(i) the right derivative operation $(Q,\cdot)$ is a left Bol quasigroup,

(ii) the left derivative operation $(Q,\cdot)$ is a right Bol quasigroup.

The next paragraph of the thesis is dedicated to WA-quasigroup automorphisms, inner permutations as automorphisms in relation to the left unit and in relation to arbitrary element: Lemma 2.2.2., Theorem 2.2.1..

Lemma 2.2.2. In a WA-quasigroup $(Q,\cdot)$ with a left identity element f, inner permutations $L_{x,y}, R_{x,y}$ and T_x relative to the element f, are automorphisms of the quasigroup $(Q,\cdot)$.

Theorem 2.2.1. In a WA-quasigroup $(Q,\cdot)$ with the left identity element f, inner permutations $L_{x,y}, R_{x,y}$ and T_x relative to $a \in Q$, are automorphisms in $(Q,\cdot)$ if and only if $a \in N_l$ and the following identity $xy \cdot a = xf \cdot ya$ is satisfied.

Pseudo-automorphisms were investigated. We know that the quasigroup with pseudo-automorphism on the right has also a unity on the left (see [31]).

Lemma 2.3.1. In a WA-quasigroup with a left identity element f, the translation R_a is a right pseudo-autmorphism if and only if the translation L_a is a right pseudo-autmorphism and $a^2 = f$.

In the fourth section of the second chapter, the properties of a new class of quasigroups introduced by Florea I. A. and the author of the thesis, namely $OWIP$-quasigroups, were researched.

Loops with weakened inverse property loops have been researched by Osborne [32]. Other properties of WIP-loops are also demonstrated in this source.

Definition 2.4.1. Quasigroup $(Q,\cdot)$ is called a $OWIP$-*quasigroup*, if in $(Q,\cdot)$ the following identity is true

$$x \cdot I(y \cdot \alpha x) = Iy, \tag{2.24}$$

for all $x, y \in Q$, where I and α are some permutations of the set Q.

From identity (2.24), by applying the properties of permutations, a new identity (2.25) was obtained for this quasigroup class: Lemma 2.4.1..

Lemma 2.4.1. In a $OWIP$-quasigroup $(Q,\cdot)$ the following identity holds:

$$I^{-1}(xz) \cdot \alpha x = I^{-1}z, \tag{2.25}$$

for any $x, z \in Q$.

By applying (2.25) the necessary and sufficient condition was found when the OWIP-quasigroup $(Q,\cdot)$ is an isotope of the LIP-loop $(Q,\circ)$. See Theorem 2.4.1..

Theorem 2.4.1. $OWIP$-quasigroup $(Q,\cdot)$ is an isotope of the LIP-loop $(Q,\circ)$, where

$$x \circ y = R_a^{-1}x \cdot L_b^{-1}y, xy = R_a x \circ L_b y \tag{2.26}$$

if and only if in $(Q,\cdot)$ the following equality is true

$$b \cdot I(I^{-1}(by) \cdot x) = R_{e_b}^{-1}\big(b \cdot I(I^{-1}b \cdot x)\big) \cdot y, \tag{2.27}$$

where $be_b = b,\ R_{e_b}v = ve_b$.

Assuming that any loop $(Q,\circ)$, isotopic of the $OWIP$-quasigroup $(Q,\cdot)$, is a LIP-loop, then in the quasigroup $(Q,\cdot)$ the folloving identity takes place:

$$z \cdot I(I^{-1}(zy) \cdot x) = R_{e_z}^{-1}\big(z \cdot I(I^{-1}z \cdot x)\big) \cdot y, \tag{2.28}$$

for any $x, y, z \in Q$ where I is a permutation of the set $Q, ze_z = z, R_{e_z}t = te_z$.

Lemma 2.4.2. If in the quasigroup $(Q,\cdot)$ the identity (2.28) takes place, then in $(Q,\cdot)$ the following identity is true :

$$z \cdot I(y \cdot \alpha z) = Iy, \tag{2.29}$$

for any $z, y \in Q$, where α is the mapping of the set Q into itself. And if α is a permutation, then $(Q,\cdot)$ is an $OWIP$-quasigroup.

The results on the right are presented analogously.

The last two sections of this chapter present the results obtained in the research of CI-quasigroups.

CI-loops are classical objects of quasigroup theory. This loop class was defined by Rafael Artzy [10]. V.D. Belousov and B.V. Tsurkan defined CI-quasigroups in [11]. Some applications of CI-quasigroups in cryptology are presented in [33].

Loop $(Q,\cdot)$ satisfying one of the equivalent identities, $x \cdot yJx = y, xy \cdot Jx = y$, where J is a bijection of the set Q such that $x \cdot Jx = 1$, is called a CI-*loop*. Quasigroup $(Q,\cdot)$ with the identity $xy \cdot Jx = y$, where J is a map of the set Q, is called CI-*quasigroup*.

Notice, in this case the maping J is a permutation of the set Q. In any CI-quasigroup the permutation J is unique ([11], Lemma 2.25).

Groupoid $(Q,\cdot)$ with the identity $xy \cdot I_r x = y$, where I_r is a maping of the set Q into itself, is called a left CI-*groupoid*. Groupoid $(Q,\cdot)$ with the identity $I_l x \cdot yx = y$, where I_l is a maping of the set Q into itself, is called a right CI-*groupoid*. Groupoid $(Q,\cdot)$ with both identities is called a CI-*groupoid*.

In fundamental article [11] the following facts are proved: any CI-groupoid is a quasigroup; in CI-quasigroup the right and left identities of CI-groupoid are equivalent; any left CI-groupoid is a left quasigroup.

From the results of Keedwell and Shcherbacov it follows that the left CI-groupoid in which the maping I_r is bijective is a CI-quasigroup. Any finite left CI-groupoid is a CI-quasigroup [34].

The main result of this section is reflected in the following theorem:

Theorem 2.5.1. Any left CI-groupoid $(Q,\cdot)$ is a CI-quasigroup.

It is clear that the similar theorem is true for any right CI-groupoid.

By researching the isotopes of these quasigroups, we obtained the following propositions:

Proposition 2.6.1. CI- quasigroup $(Q,\cdot)$ is the isotope of the group $(Q,\circ)$, with the isotopy (2.26) if and only if in $(Q,\cdot)$ the following equality is true

$$\big(x(y(zu))\big) \cdot v = y \cdot \big((xz \cdot v) \cdot u\big), \tag{2.36}$$

for any $x, y, z, u, v \in Q$.

To the question, if in the arbitrary quasigroup $(Q,\cdot)$ the identity (2.36) is fulfilled, is it $(Q,\cdot)$ a CI-quasigroup, we answer by Proposition 2.6.2..

Proposition 2.6.2. If in the arbitrary quasigroup $(Q,\cdot)$ the identity (2.36) is true, then $(Q,\cdot)$ is a CI-quasigroup.

Proposition 2.6.3. If any loop $(Q,\circ)$ isotopic to CI-quasigroup $(Q,\cdot)$, is commutative, then the quasigroup $(Q,\cdot)$ is medial, and $(Q,\circ)$ is abelian group.

Proposition 2.6.4. Loop $(Q,\circ)$, isotopic to CI-quasigroup $(Q,\cdot)$, where the isotopy is given by the equality (2.26) will be CI-loop if and only if in $(Q,\cdot)$ the folloving is true

$$(x \cdot by)a = (x \cdot ba)y, \tag{2.41}$$

for any $x, y \in Q$.

Moving on to the quasigroup $(Q,\cdot)$ study in which equality (2.41) takes place $\forall a,b,x,y \in Q$, we have got:

Proposition 2.6.5. Any quasigroup $(Q,\cdot)$ with identity

$$(x \cdot yz)t = (x \cdot yt)z, \tag{2.44}$$

$\forall x,y,z,t \in Q$ is a medial CI-quasigroup.

In the **third Chapter – About a class of *i*-quasigroups. Units in Bol-Moufang quasigroups –** consisting of six sections, objectives 2 and 3 are achieved. The results are published in [13], [27], [24], [28], [18].

In this chapter a new class of quasigroups called i-quasigroups introduced by the first aspirant of V.D. Belousov, I.A. Florea is researched.

In the first section of this chapter, the distributant of i-quasigroups was researched. *Distributant* D of quasigroup $(Q,\cdot)$ consists of all elements d of the set Q such that $(x \cdot y) \cdot d = (x \cdot d) \cdot (y \cdot d), d \cdot (x \cdot y) = (d \cdot x) \cdot (d \cdot y)$ for any $x,y \in Q$.

Definition 3.1.2. Quasigroup $(Q,\cdot)$ is caleed an i-*quasigroup*, if in $(Q,\cdot)$ the following identity is true:

$$x(xy \cdot z) = y(zx \cdot x), \tag{3.1}$$

where $x,y,z \in Q$.

The obtained results are formulated in the following theorems:

Theorem 3.1.1. If i -quasigroup $(Q,\cdot)$ is an RIP-quasigroup, then $(Q,\cdot)$ is a quasigroup Moufang with a left identity element f and distributant $D = \{f\}$.

Theorem 3.1.2. If i-quasigroup $(Q,\cdot)$ with left unit f is an isotop of abelian group, then $(Q,\cdot)$ is a medial Moufang quasigroup and distributant $D = \{f\}$.

Theorem 3.1.3. If distributant D of i-quasigroup $(Q,\cdot)$ is nonempty and $(Q,\cdot)$ is an isotop of left Bol loop, then $(Q,\cdot)$ is a left Bol quasigroup.

Theorem 3.1.4. i-quasigroup $(Q,\cdot)$ with nonempty distributant D is a left Bol quasigroup if and only if in $(Q,\cdot)$ the following equality is true:

$$xa \cdot xy = xx \cdot ay, \tag{3.6}$$

for any $x,y \in Q$, where $a \in D, a$ is a fixed element.

The relationships of i-quasigroups with other classes of quasigroups were researched by applying different conditions. We see the results obtained in the following propositions:

Proposition 3.2.1. If i-quasigroup $(Q,\cdot)$ is an idempotent, then $(Q,\cdot)$ is a left Bol quasigroup. Moreover, in this case, $(Q,\cdot)$ is right Stein quasigroup, which is a left distributive, and core of this left Bol quasigroup is an i-quasigroup, too.

Proposition 3.2.2. If i-quasigroup $(Q,\cdot)$ has right identity element e, then $(Q,\cdot)$ is a Moufang loop in wich $x^2y = yx^2$ for any $x, y \in Q$.

Proposition 3.2.3. Any i-quasigroup $(Q,\cdot)$ with left identity element f is a quasigroup with left inverse property and isotopic to of LIP-loop $(Q,\circ)$, where $x \circ y = R_f^{-1}x \cdot y$.

Proposition 3.2.4. If in i-quasigroup $(Q,\cdot)$ the equality $x^2 = f$ takes place for any $x \in Q$, where f is a fixed element, then $(Q,\cdot)$ is a Moufang quasigroup with left unit element f and an isotope of abelian group.

Proposition 3.2.5. i- quasigroup $(Q,\cdot)$ with left identity element f is a Moufang quasigroup if and only if R_f is an automorphism of quasigroup $(Q,\cdot)$.

In the third section of this chapter, the quasigroups with the following three laws were researched:

$$x \cdot xz = xx \cdot z \ (left\ alternative\ identity), \tag{3.19}$$

$$zx \cdot x = z \cdot xx \ (right\ alternative\ identity), \tag{3.20}$$

$$xy \cdot x = x \cdot yx \ (identity\ of\ elasticity). \tag{3.21}$$

They take place Propositions 3.3.1.-3.3.3.. Let us highlight:

Proposition 3.3.3. In every i-quasigroup $(Q,\cdot)$ with identity of elasticity (3.21) a set of all local unit elements forms left Bol subquasigroup.

Proposition 3.3.4. i-quasigroup $(Q,\cdot)$ with left identity element f is a Moufang quasigroup if and only if in $(Q,\cdot)$ the folloving identity is true:

$$zx \cdot x = zf \cdot xx, \tag{3.26}$$

for any $x, z \in Q$.

The pseudo-automorphisms of this class of quasigroups were researched. By applying various conditions, the following results were obtained. They are presented in the following 3 propositions:

Proposition 3.4.1. If permutation α of set Q of i-quasigroup $(Q,\cdot)$ with left identity element f is a pseudo-automorphism from the right of quasigroup $(Q,\cdot)$ with companion k, then k is a left Bol element.

In Example 3.4.1. pseudo-automorphism of i-quasigroup is constructed.

Proposition 3.4.2. If in i-quasigroup $(Q,\cdot)$ with left unit f the translations L_a and R_b are right pseudo-automorphisms with companion k, then $a = e_k f, b = e_k$, where $ke_k = k$.

Proposition 3.4.3. If i-quasigroup $(Q,\cdot)$ with left unit f is an isotope of abelian group, then L_a and R_b,where $a = e_k f, b = e_k$, are right pseudo-automorphisms of the quasigroup $(Q,\cdot)$ with companion k.

The penultimate section of this chapter is dedicated to the research of 60 classical Bol-Moufang type identities, with the existence of the unit on the right, on the left. Whe use list of 60 Bol-Moufang type identities given in [35]. See Table 3.1 [36].

There exist other definitions of Bol-Moufang type identities and, thereofore, other lists and classifications of such identities [37], [38].

Reformulating the title of Gagola`s article “How and why Moufang loops behave like a group” we can say that quasigroups with Bol-Moufang identities “behave like a group”. This is one of reasons why we study these quasigroup classes.

We recall, (12)-parastroph of groupoid $(G,\cdot)$ is a groupoid $(G,*)$ in which operation " $*$ " is obtained by the following rule:

$$x * y = y \cdot x. \tag{3.44}$$

It is clear that for any groupoid $(G,\cdot)$ there exists its (12)-parastroph groupoid $(G,*)$.

Suppose that an identity F is true in groupoid $(G,\cdot)$. Then we can obtain (12)-parastrophic identity (F^*) of the identity F replacing the operation " $\cdot$ " on the operation " $*$ " and changing the order of variables using rule (3.44).

Remark 3.5.1. In quasigroup case, similarly to (12)-parastrophe identity, other parastrophic identities can be defined.

Table 3.1. Identity elements in quasigroups with Bol-Moufang identities.

Name	Abbrev.	identity	f	e	loop	group
F_1		$xy \cdot zx = (xy \cdot z)x$	+	+	+	+
F_2	middle Moufang	$xy \cdot zx = (x \cdot yz)x$	+	+	+	-
F_3		$xy \cdot zx = x(y \cdot zx)$	+	+	+	+
F_4	middle Moufang	$xy \cdot zx = x(yz \cdot x)$	+	+	+	-
F_5		$(xy \cdot z)x = (x \cdot yz)x$	+	+	+	+
F_6	extra identity	$(xy \cdot z)x = x(y \cdot zx)$	+	+	+	-
F_7		$(xy \cdot z)x = x(yz \cdot x)$	+	-	-	-
F_8		$(x \cdot yz)x = x(y \cdot zx)$	-	+	-	-
F_9		$(x \cdot yz)x = x(yz \cdot x)$	-	-	-	-
F_{10}		$x(y \cdot zx) = x(yz \cdot x)$	+	+	+	+
F_{11}		$xy \cdot xz = (xy \cdot x)z$	+	+	+	+
F_{12}		$xy \cdot xz = (x \cdot yx)z$	+	+	+	+

F_{13}	extra identity	$xy \cdot xz = x(yx \cdot z)$	+	+	+	-
F_{14}		$xy \cdot xz = x(y \cdot xz)$	+	+	+	+
F_{15}		$(xy \cdot x)z = (x \cdot yx)z$	-	-	-	-
F_{16}		$(xy \cdot x)z = x(yx \cdot z)$	+	-	-	-
F_{17}	left Moufang	$(xy \cdot x)z = x(y \cdot xz)$	+	+	+	-
F_{18}		$(x \cdot yx)z = x(yx \cdot z)$	+	+	+	+
F_{19}	left Bol	$(x \cdot yx)z = x(y \cdot xz)$	-	+	-	-
F_{20}		$x(yx \cdot z) = x(y \cdot xz)$	+	+	+	+
F_{21}		$yx \cdot zx = (yx \cdot z)x$	+	+	+	+
F_{22}	extra identity	$yx \cdot zx = (y \cdot xz)x$	+	+	+	-
F_{23}		$yx \cdot zx = y(xz \cdot x)$	+	+	+	+
F_{24}		$yx \cdot zx = y(x \cdot zx)$	+	+	+	+
F_{25}		$(yx \cdot z)x = (y \cdot xz)x$	+	+	+	+
F_{26}	right Bol	$(yx \cdot z)x = y(xz \cdot x)$	+	-	-	-
F_{27}	right Moufang	$(yx \cdot z)x = y(x \cdot zx)$	+	+	+	-
F_{28}		$(y \cdot xz)x = y(xz \cdot x)$	+	+	+	+
F_{29}		$(y \cdot xz)x = y(x \cdot zx)$	-	+	-	-
F_{30}		$y(xz \cdot x) = y(x \cdot zx)$	-	-	-	-
F_{31}		$yx \cdot xz = (yx \cdot x)z$	+	+	+	+
F_{32}		$yx \cdot xz = (y \cdot xx)z$	+	+	+	+
F_{33}		$yx \cdot xz = y(xx \cdot z)$	+	+	+	+
F_{34}		$yx \cdot xz = y(x \cdot xz)$	+	+	+	+
F_{35}		$(yx \cdot x)z = (y \cdot xx)z$	-	+	-	-
F_{36}	RC-identity	$(yx \cdot x)z = y(xx \cdot z)$	+	-	-	-
F_{37}	C-identity	$(yx \cdot x)z = y(x \cdot xz)$	-	-	-	-
F_{38}		$(y \cdot xx)z = y(xx \cdot z)$	+	+	+	-
F_{39}	LC-identity	$(y \cdot xx)z = y(x \cdot xz)$	-	+	-	-
F_{40}		$y(xx \cdot z) = y(x \cdot xz)$	+	-	-	-
F_{41}	LC-identity	$xx \cdot yz = (x \cdot xy)z$	+	+	+	-
F_{42}		$xx \cdot yz = (xx \cdot y)z$	+	-	-	-
F_{43}		$xx \cdot yz = x(x \cdot yz)$	+	-	-	-
F_{44}		$xx \cdot yz = x(xy \cdot z)$	+	-	-	-

F_{45}		$(x \cdot xy)z = (xx \cdot y)z$	+	-	-	-
F_{46}	LC-identity	$(x \cdot xy)z = x(x \cdot yz)$	-	-	-	-
F_{47}		$(x \cdot xy)z = x(xy \cdot z)$	+	+	+	+
F_{48}	LC-identity	$(xx \cdot y)z = x(x \cdot yz)$	+	-	-	-
F_{49}		$(xx \cdot y)z = x(xy \cdot z)$	+	-	-	-
F_{50}		$x(x \cdot yz) = x(xy \cdot z)$	+	+	+	+
F_{51}		$yz \cdot xx = (yz \cdot x)x$	-	+	-	-
F_{52}		$yz \cdot xx = (y \cdot zx)x$	-	+	-	-
F_{53}	RC-identity	$yz \cdot xx = y(zx \cdot x)$	+	+	+	-
F_{54}		$yz \cdot xx = y(z \cdot xx)$	-	+	-	-
F_{55}		$(yz \cdot x)x = (y \cdot zx)x$	+	+	+	+
F_{56}	RC-identity	$(yz \cdot x)x = y(zx \cdot x)$	-	-	-	-
F_{57}	RC-identity	$(yz \cdot x)x = y(z \cdot xx)$	-	+	-	-
F_{58}		$(y \cdot zx)x = y(zx \cdot x)$	+	+	+	+
F_{59}		$(y \cdot zx)x = y(z \cdot xx)$	-	+	-	-
F_{60}		$y(zx \cdot x) = y(z \cdot xx)$	-	+	-	-

It is clear that an identity F is true in groupoid $(G,\cdot)$ if and only if in groupoid $(Q,*)$ identity F^* is true.

We highlight Theorem 3.5.1., on the basis of which the statements for the (12)-parastrophic identities of the investigated quasigroups were proved.

Theorem 3.5.1. [36]. $(F_1)^* = F_3, (F_2)^* = F_4, (F_5)^* = F_{10}, (F_6)^* = F_6, (F_7)^* = F_8, (F_9)^* = F_9,$
$(F_{11})^* = F_{24}, (F_{12})^* = F_{23}, (F_{13})^* = F_{22}, (F_{14})^* = F_{21}, (F_{15})^* = F_{30}, (F_{16})^* = F_{29},$
$(F_{17})^* = F_{27}, (F_{18})^* = F_{28}, (F_{19})^* = F_{26}, (F_{20})^* = F_{25}, (F_{31})^* = F_{34}, (F_{32})^* = F_{33},$
$(F_{35})^* = F_{40}, (F_{36})^* = F_{39}, (F_{37})^* = F_{37}, (F_{38})^* = F_{38}, (F_{41})^* = F_{53}, (F_{42})^* = F_{54},$
$(F_{43})^* = F_{51}, (F_{44})^* = F_{52}, (F_{45})^* = F_{60}, (F_{46})^* = F_{56}, (F_{47})^* = F_{58}, (F_{48})^* = F_{57},$
$(F_{49})^* = F_{59}, (F_{50})^* = F_{55}$.

Considering Lemma 3.5.1. and 3.5.4., we solve the problem of the existence of units in the classical Bol-Moufang type identities.

We highlight some theorems and corollaries:

Theorem 3.5.6. Quasigroup $(Q,\cdot)$ with identity F_7 $(xy \cdot z)x = x(yz \cdot x)$ has a right and has no left unit.

Corollary 3.5.2. Quasigroup $(Q,\cdot)$ with identity F_8 $(x \cdot yz)x = x(y \cdot zx)$ has a left and has no right unit.

Theorem 3.5.7. Quasigroup $(Q,\cdot)$ with identity F_9 $(x \cdot yz)x = x(yz \cdot x)$ does not have left and right units.

Theorem 3.5.9. Quasigroup $(Q,\cdot)$ with identity F_{12} $xy \cdot xz = (x \cdot yx)z$ is a group.

Corollary 3.5.4. Quasigroup $(Q,\cdot)$ with identity F_{23} $yx \cdot zx = y(xz \cdot x)$ is a group.

Theorem 3.5.21. Quasigroup $(Q,\cdot)$ with identity F_{36} $(RC\ identity)$ $(yx \cdot x)z = y(xx \cdot z)$ has a left and has no right unit.

Corollary 3.5.16. Quasigroup $(Q,\cdot)$ with identity F_{39} $(LC\ identity)$ $(y \cdot xx)z = y(x \cdot xz)$ has a right and has no left unit.

Theorem 3.5.24. Quasigroup $(Q,\cdot)$ with identity F_{41} $(LC\ identity)$ $xx \cdot yz = (x \cdot xy)z$ is a loop.

Corollary 3.5.17. Quasigroup $(Q,\cdot)$ with identity F_{53} $(RC\ identity)$ $yz \cdot xx = y(zx \cdot x)$ is a loop.

This solves objective 2.

In **Chapter four – Left-transitive quasigroup. Neumann and Schweizer quasigroups –** the results published in the following papers are reflected: [20], [21], [25], [26], [16], [17]. In this chapter the objectives are achieved, which refer to the research of the G-properties of the left - transitive quasigroups and Neumann and to the research of the relations of these quasigroups with the Moufang quasigroups, left Bol (right), and other quasigroup classes.

Quasigroup $(Q,\cdot)$ is said to be *left-transitive* if in this quasigroup the identity $xy \cdot xz = yz$ holds. Quasigroup $(Q,\cdot)$ is said to be *Neumann quasigroup* if in this quasigroup the identity $(Q,\cdot)$ $x \cdot (yz \cdot yx) = z$ holds true.

In sections 4.1.-4.5. the results obtained in the research of the left-transitive quasigroups are formulated, namely: about the relationship of these quasigroups with other classes of quasigroups, about the nuclei, about morphisms, about G-properties.

In sections 4.6.-4.8. analogously, the results obtained for the Neumann quasigroups are formulated.

Both classes are isotopic to the group. We have theorems:

Theorem 4.1.1. Any left-transitive quasigroup $(G,\circ)$ can be obtained from a group $(G,+)$ (not necessary commutative) using the following construction:

$$x \circ y = -x + y = Ix + y, \tag{4.2}$$

where $x + Ix = 0$ for all $x, y \in G$.

Theorem 4.6.3. Any Neumann quasigroup $(Q,\cdot)$ is isotopic of an abelian group $(Q,+)$ of the form $x \cdot y = x - y$.

By researching the nuclei of these quasigroups we obtained: left nucleus $(N_l,\cdot)$ of left transitive quasigroup $(Q,\cdot)$ is a normal subgroup of quasigroup $(Q,\cdot)$ with consists of elements of order two that lie in the centre of group $(Q,+)$; right nucleus of Neumann quasigroup $(Q,\cdot)$ contains such elements of the set Q, such that $a = -a$.

By researching autotopies and quasiautomorphisms, their forms were obtained. By researching the G-properties of these quasigroups, we obtained that any Neumann quasigroup $(Q,\cdot)$ is a GA-quasigroup (Theorem 4.8.1.); for the left-transitive quasigroup the necessary and sufficient condition is given that it to be a GA-quasigroup (Corollary 4.5.1.).

Quasigroup $(Q,\cdot)$ with identity $yz \cdot yx = xz$ is called a *Schweizer quasigroup.*

The important result is that the Neumann quasigroup class coincides with the Schweizer quasigroup class (Theorem 4.6.5.).

Theorem 4.6.5. Any Schweizer quasigroup $(Q,\cdot)$ is a Neumann quasigroup and wise versa.

Thanks. I express my sincere thanks to the scientific leader *Victor Alexeevich Shcerbacov* for determining the field of research, for formulating research objectives, for the knowledge I gained during the four years of my doctorate, for the help he gave me in making the publications and forming the thesis.

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5. GENERAL CONCLUSIONS

Research conducted in the doctoral thesis “**Morphisms and properties of non-associative algebraic systems with Moufang conditions**” fully correspond to the purpose and objectives set out in the introduction.

The quasigroup class contains that of the groups (any group is a quasigroup), a field that experienced an extraordinary development in the twentieth century and continues to develop rapidly today. The notion of quasigroup is more general than that of group, so it is found as an algebraic equivalent in a wider range of environmental problems in which we exist and requires specific approaches, which are missing in group theory.

The important scientific problem solved is the application of morphism relations to the research of properties and units of non-associative algebraic systems with Moufang-type conditions leading to the description of important new relations between quasigroup classes.

The main results of the paper are new. The analysis of the obtained results allows us to highlight the following general results:

1. It has been established that any WA-quasigroup, which is IP-quasigroup, is a Moufang quasigroup;

   It turned out that in the WA-quasigroup with left unit the internal permutations in relation to the unit are automorphisms of the quasigroup; for internal permutations with respect to the element $a \in Q$ the necessary and sufficient condition was found when there are automorphisms [14];
2. By researching of generalized WIP-quasigroups it was found the condition when this quasigroup is an isotope of loop with left inverse property. Assuming that any isotope loop $OWIP$-quasigroup is LIP-loop, we obtained a new identity in this quasigroup, for which it was found the relation with the left(right) Bol quasigroup [22];
3. In the research of CI-quasigroups the main result is that any left CI-groupoid $(Q,\cdot)$ is a CI-quasigroup [19], [15];
4. Defining a new class of quasigroups (i-quasigroups) their relationships with other classes of quasigroups were researched. Assuming that the i-quasigroup $(Q,\cdot)$ is idempotent, it turned out to be the left Bol quasigroup, right Stein quasigroup [13], [24];
5. The problem of the existence of the unit (left, right, middle) in quasigroups with the Bol-Moufang type identities, listed in the paper Extra loops II, by F. Fenyves (1969), has been solved. The paper presents a table with information on the existence of the unit for each of the 60 identities [18], [27];
6. It has been shown that the notion of Neumann quasigroup coincides with that of Schweizer quasigroup [20], [21], [26], [16].

The thesis proposed for defence contains the complete solution of the problem in the application of morphism relations to the research of properties and units of non-associative algebraic systems with Bol-Moufang type conditions leading to the description of important new relations between quasigroup classes.

Recommendation:

1. The units were described for each of the 60 quasigroups that satisfy the classical Bol-Moufang type identities. This information will be useful in further research of quasigroups with Bol-Moufang identities.

2. The result that any CI-groupoid is a CI-quasigroup opens up new possibilities in the research of CI-quasigroups.
3. Two new classes of quasigroups are defined in the thesis: i-quasigroups and $OWIP$-quasigroups. The paper investigates some properties of these classes of quasigroups, but their general theory is to be developed.
4. It is recommended that the results obtained be applied to the development of optional courses for master's and doctoral students.

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SUMMARY

In the thesis "**Morphisms and properties of non-associative algebraic systems with Moufang type conditions**", submitted by Diduric Natalia for obtaining the title of doctor in mathematical sciences in the specialty 111.03 - Mathematical Logic, Algebra and Number Theory. The thesis was developed at the State University of Moldova, Chisinau, 2021.

Thesis structure: the thesis is written in Romanian and contains an introduction, four chapters, general conclusions and recommendations, 98 bibliographic titles, 95 pages (including 86 pages of basic text). The obtained results are published in 16 scientific papers.

Keywords: quasigroup, loop, group, groupoid, isotope, automorphism, left unit, right unit, pseudo-automorphism, left Bol (right) quasigroup, Moufang quasigrup, WA-quasigroup, CI-quasigroup, i-quasigroup, medial quasigroup, Neumann quasigroup, transitivity, G-properties.

Thesis field of study: algebra, more precisely, the theory of quasigroups with identities including Bol-Moufang-type identities, properties of non-associative algebraic systems.

The purpose and objectives of the paper. The aim of the paper is to investigate the properties of non-associative algebraic systems with Bol-Moufang type identities. To achieve this goal, the following objectives have been defined: research on the relations of WA-, CI-quasi-groups, transitive on the left and Neuman with the quasigroups Moufang, Bol on the left, on the right, etc.; research of quasigroups with any of the 60 classical Bol-Moufang identities listed in [12] at the existence of the unit; research of morphisms, properties, relationships with other classes of quasigroups of newly defined quasigroups (i-quasigroups and WIP-generalized quasigroups); research on the G-properties of left transitive quasigroups and Neumann.

Scientific novelty and originality consist in obtaining new theoretical results. All the results presented in the thesis are new and original. Classes of quasigroups already researched (WA-, CI-quasigroups, transitive left quasigroups, Neumann, etc.) were researched. Two new classes of quasigroups were introduced and researched (i-quasigroups, WIP-generalized quasigroups). Quasigroup classes isotopic to groups were investigated. The properties of some classes of invertible quasigroups are described. Connections between the studied quasigroup classes and the classical quasigroups Moufang, Bol, etc. were investigated. The general forms of the automorphisms, pseudo-automorphisms and quasitomorphisms of these quasigroups were determined.

ADNOTARE

La teza “**Morfismele şi proprietăţile sistemelor algebrice neasociative cu condiţii de tip Moufang**”, înaintată de către Diduric Natalia pentru obţinerea titlului de doctor în ştiinţe matematice la specialitatea 111.03 - Logică Matematică, Algebră şi Teoria Numerelor.

Teza a fost elaborată la Universitatea de Stat din Moldova, Chişinău, anul 2021.

Structura tezei: teza este scrisă în limba română şi conţine introducere, patru capitole, concluzii generale şi recomandări, 98 titluri bibliografice, 95 pagini (inclusiv 86 pagini de text de bază). Rezultatele obţinute sunt publicate în 16 lucrări ştiinţifice.

Cuvinte-cheie: cvazigrup, buclă, grup, grupoid, izotop, automorfism, unitate la stânga, unitate la dreapta, pseudoautomorfism, cvazigrup Bol la stânga (la dreapta), cvazigrup Moufang, WA-cvazigrup, CI-cvazigrup, i-cvazigrup, cvazigrup medial, cvazigrup Neumann, cvazigrup tranzitiv, G-proprietăţi.

Domeniul de studiu al tezei: algebră, în special, teoria cvazigrupurilor cu identităţi, inclusiv identităţile de tip Bol-Moufang, proprietăţile sistemelor algebrice neasociative.

Scopul şi obiectivele lucrării. Scopul lucrării este cercetarea proprietăţilor sistemelor algebrice neasociative cu identităţi de tip Bol-Moufang. Pentru atingerea acestui scop au fost definite următoarele obiective: cercetarea relațiilor WA-, CI-cvazigrupurilor, cvazigrupurilor tranzitive la stânga și Neumann cu cvazigrupurile Moufang, Bol la stânga, Bol la dreapta ș.a.; cercetarea existenței unității în cvazigrupurile cu fiecare dintre cele 60 de identități de tip Bol-Moufang, enumerate în [12]; cercetarea morfismelor, proprietăților, relațiilor cu alte clase de cvazigrupuri noi definite în lucrare (i-cvazigrupuri și $OWIP$-cvazigrupuri); cercetarea G-proprietăților cvazigrupurilor tranzitive la stânga și Neumann.

Noutatea şi originalitatea ştiinţifică constă în obţinerea rezultatelor noi de ordin teoretic. Toate rezultatele prezentate în teză sunt noi şi originale. Au fost cercetate diverse clase de cvazigrupuri (WA-, CI-cvazigrupuri, cvazigrupuri tranzitive la stânga, Neumann ș.a.). Au fost introduse şi cercetate două clase noi de cvazigrupuri (WIP-cvazigrupuri generalizate, i-cvazigrupuri). Au fost cercetate clase de cvazigrupuri izotope grupurilor. Sunt descrise proprietăţile unor clase de cvazigrupuri inversabile. Au fost cercetate conexiuni între clasele de cvazigrupuri studiate și cvazigrupurile clasice Moufang, Bol ș.a. Sunt determinate formele generale ale automorfismelor, pseudoautomorfismelor și cvaziautomorfismelor acestor cvazigrupuri.

MORPHISMS AND PROPERTIES OF NON-ASSOCIATIVE ALGEBRAIC SYSTEMS WITH MOUFANG TYPE CONDITIONS

111.03 MATHEMATICAL LOGIC, ALGEBRA AND NUMBER THEORY

Summary of Ph.D. Thesis in Mathematics

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