MOLDOVA STATE UNIVERSITY

DOCTORAL SCHOOL OF PHYSICAL, MATHEMATICAL, INFORMATION AND ENGINEERING SCIENCES

As a manuscript
C.Z.U.: 512.548

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MORPHISMS AND PROPERTIES OF NON-ASSOCIATIVE ALGEBRAIC SYSTEMS WITH MOUFANG TYPE CONDITIONS

Speciality 111.03 – mathematical logic, algebra, and number theory

PhD in Mathematical Sciences thesis summary

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CHISINAU, 2022

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The thesis defense will take place on March 29, 2022, time 14-00, „Vladimir Andrunachievici" Institute of Mathematics and Computer Science, Academiei str. 5, room 340..

The summary and Phd thesis can be consulted at the library of the Moldova State University and on the ANACEC website (www.cnaa.md).

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CONTENTS

1. CONCEPTUAL REFERENCES OF RESEARCH

The topicality and importance of the problem addressed. Quasigroup theory began in the 20-30s of the XX century when, after the fundamental works of David Hilbert in the late nineteenth century on the axiomatization of mathematics and, in particular, on the axiomatization of geometry, there have appeared works related to different axiom systems, in the main, to the axiom systems of various geometries, including Euclidean geometry, projective geometry, Lobachevsky geometry, mostly, in dimensions 2 and 3.

The very term "quasigroup" appeared in Ruth Moufang's paper [1] on the coordinatization of projective planes. We can also say that the term "quasigroup" appeared when studying the question of the independence of axioms in the axiom systems of the projective plane. In her works, Moufang understood by quasigroup the object, which is now called a Moufang loop. In modern terms, she defined the Moufang loop as an IP-loop $(Q,\cdot)$ with some "weak associativity" identity.

The next most important paper on the subject of quasigroups appeared two years after Moufang′s one: "Gewebe und Gruppen" by Gerrit Bol (1937). Bol′s approach is from a web-geometrical point of view. He constructs three new configurations, U1, U2, U3, and asks whether the closure of these three figures implies associativity. He answers that question negatively and shows that the three U figures together imply only the law $a\,[b\,(cb)] = [(ab)\,c]\,b$, which is precisely one of the Moufang identities. Further, Bol explains the algebraic meaning of each of the U figures and shows that U1 and U2 correspond to laws that we now call the right Bol and the left Bol identities.

In addition, there were already several American publications on quasigroups: (1937) "Theory of quasigroups", by Hausmann and Ore; (1939) "Quasigroups, which verify certain generalized associative laws", by Murdoch; (1940) "Quasigroups", by Garrison.

All three authors have already used the term "quasigroup" in a broader sense, as we use it now, and not just as Moufang's Q^* system.

The first publications to introduce the term "loop" were the two most important works that Albert wrote in 1943: Quasigroups. I and Quasigroups. II [2, 3]. We also mention works by K. Toyoda [4], R. H. Bruck [5], R. Baer [6, 7].

To date, Moufang loops and Bol loops are the most studied ones in the quasigroup theory. In general, the middle Bol loops have been studied. Papers published by T. G. Jaiyéolá, S. P. David, and O. O. Oyebola (2021); A. O. Abdulkareem and J. O. Adeniran (2020) are dedicated to researching these loops. In the work of P. Sârbu and I. Grecu, it was shown that the commutative loops with invariant flexibility under the isostrophy of the loops are Moufang loops. V. I. Onoi, and L. A. Ursu, using the isotopic approach, generalized the concept of the Moufang binary loop in case n-ar ($n > 2$).

In the 30s of the twentieth century, the notion of a network was introduced. In the terminology of network theory, the notion of quasigroup has a clear geometric interpretation [8].

For quasigroups, especially considering their relations to combinatorics, various morphisms have been determined and actively researched, among which we mention isomorphisms, automorphisms, isotopes, autotopies, isostrophies, autostrophies, pseudo-automorphisms, generalized isotopes. Automorphisms and groups of loop automorphisms were investigated by A. A. Albert already in the first works in quasigroup theory [2]. The concept of disaster, in general, was introduced by A. Sade in France in the 1950s. Pseudo-automorphisms have been defined and researched by R. H. Bruck in the work "Pseudo-automorphisms and Moufang loops", 1952.

Definition: The G-loop is the isomorphic loop of all its isotope loops. It has a geometric origin. R. H. Bruck, in his 1971 paper, pointed out that an unsolved problem is the problem of describing G-loops. Geometrically, Bruck's problem was solved by Barlotti and Strambach. Belousov solved this problem and proved that the loop $(L,\cdot)$ is a G-loop if and only if any element of it is the companion of one pseudoautomorphism to the right and to the left of the loop $(L,\cdot)$ ([8], Theorem 3.8). V. D. Belousov's results pave the way for research into G-properties, i.e. research into G-loops and G-quasigroups. In recent years, G-loop theory has developed very actively. New classes of G-loops are discovered. The problem of researching left-right and middle-class GA-quasigroups remains unresolved.

In the Republic of Moldova, V. Belousov (1925-1988) and his disciples (Florja, Basarab, Gvaramia, etc.) obtained fundamental results in quasigroup theory. The Moufang quasigroups, which are currently being studied extensively, have been defined and researched by V. Belousov. In his work, W. A. Dudek described the linear shape of these quasigroups and characterized their parastrophes. I. A. Florja introduced in 1965 a special class of quasigroups, which we call Bol quasigroups [9]. In 1976 he researched the relationship of left-transitive quasigroups with Bol quasigroups [10]. The problem arises of prolonging the research of the relations of some known classes of quasigroups with the Moufang, on the left (on the right) Bol quasigroups, etc.

Almost all well-known (classical) classes of quasigroups and loops possess the property of invertibility. Most often these quasigroups possess one of the properties of invertibility, namely *IP*, *LIP*, *RIP*, *WIP*, or *CI*. IP- and LIP-loops were defined in the work of R. Moufang [1]. WIP-loops - in the work of R. Baer [6], and CI-loops were defined by Rafael Artzy [11]. V. D. Belousov and B. V. Tsurkan defined and studied CI-quasigroups [12]. There arises the problem of continuing the study of the properties of quasigroups with the property of invertibility (WA-, CI-quasigroups).

In the work of F. Fenyves, "Extra loops II. On loops with identities of Bol-Moufang type ", (1969) 60 identities were enumerated, called the Bol-Moufang type. They include the identities Bol on the left, Bol on the right, and the four identities Moufang. These identities represent a weaker form of associative law, and in the case of loops, some of them involve associativity. Kenneth Kunen in his article "Quasigroups, Loops, and Associative Laws" (1996) asks: what quasigroup with a weak form of associative law is a loop. In particular, it completely solves the problem for all Bol-Moufang identities. There arises the problem of prolonging the research of the existence of unilateral units in quasigroups with Bol-Moufang identities.

Papers published by R. Moufang, G. Bol, R. H. Bruck, V. D. Belousov, K. Kunen, S. Gagola III, J. D. Philips, etc. are dedicated to researching quasigroups and loops with Bol-Moufang-type identities.

The following issues are raised in this thesis:

Problem 1. To research the morphisms, properties, relationships with other classes of quasigroups of newly defined quasigroups (*i*-quasigroups and *OWIP*-quasigroups).

Problem 2. To research the existence of the unilateral unit in quasigroups with Bol-Moufang-type identities, listed in the paper by F. Fenyves, "Extra loops II. On loops with identities of Bol-Moufang type", (1969).

Problem 3. To research the *G*-properties of left transitive quasigroups and Neumann quasigroup.

We mention only a few works in this direction [13, 14, 15, 16, 17, 18, 19, 20, 21, 22]. Problems 1-3 are covered in Chapters 2, 3, and 4.

The purpose and objectives of the thesis. The thesis aims to investigate the morphisms and properties of non-associative algebraic systems with Moufang-type identities. To achieve this goal, the following objectives have been defined:

(1) Research on the relations of *WA*-, *CI*-quasigroups, transitive on the left and Neumann ones with the quasigroups Moufang, Bol on the left, on the right, etc;
(2) Investigation of the existence of the unilateral unit in quasigroups with Bol-Moufang-type identities, listed in the paper by F. Fenyves, "Extra loops II. On loops with identities of Bol-Moufang type", (1969);
(3) Research of morphisms, properties, relationships with other classes of quasigroups of newly defined quasigroups (*i*-quasigroups and *OWIP*-quasigroups);
(4) Research of *G*-properties of left transitive quasigroups and Neumann quasigroup.

The scientific novelty and originality consist in obtaining new theoretical results with applications in other fields. All the results presented in the thesis are new and original. Different quasigroup classes (*WA*-, *CI*-quasigroups, left transitive quasigroups, Neumann quasigroups) were researched. Two new classes of quasigroups (*OWIP*-quasigroups, *i*-quasigroups) were introduced and researched. Classes of quasigroup isotopic to a group were investigated. The properties of some classes of invertible quasigroups are described. The connections of the quasigroup classes investigated with the classical quasigroups Moufang, Bol, and other quasigroup classes were researched. The forms of automorphisms, pseudo-automorphisms, and quasiautomorphisms of these quasigroups were determined.

The important scientific problem solved consists in the research of different morphisms (autotopies, pseudoautomorphisms, *G*-properties) and notions (distributant, nucleus) in non-associative algebraic systems with Bol-Moufang conditions that lead to the description of important new relationships between the classes studied by quasigroups (including newly introduced classes).

The theoretical importance and applicative value of the thesis are determined by obtaining new results in the research of non-associative systems of the Bol-Moufang type. The paper is of theoretical character. The methods developed in the paper allowed solving the problems.

Approval of results. The scientific results obtained were presented and approved during the Special Session of the Seminar "Algebra and Mathematical Logic", dedicated to the memory of Professor V. Belousov, the Vladimir Andrunachievici Institute of Mathematics and Computer Science of Moldova. The main results included in the thesis were presented at the following scientific conferences:

- "Tendințe contemporane ale dezvoltării științei: viziuni ale tinerilor cercetători": Conferința Științifică a Doctoranzilor (cu participare internațională), ediția a 6-a, Chișinău, 15, June 2017.
- The Fourth Conference of Mathematical Society of the Republic of Moldova: dedicated to the centenary of Vladimir Andrunachievici (1917-1997): Proceedings CMSM 4, June 28 – July 2, 2017, Chișinău.
- The 25 Conference on Applied and Industrial Mathematics CAIM 2017, September 14–17, 2017, Iași.

- International conference on mathemathics, informathics, and information technologies dedicated to the illustrious scientist Valentin Belousov, 19-21 april, 2018, Bălţi.
- “Tendinţe contemporane ale dezvoltării științei: viziuni ale tinerilor cercetători”: Conferința Științifică a Doctoranzilor (cu participare internațională), ediția a VII-a Chișinău, 15, June 2018.
- The 26th Conference on Applied and Industrial Mathematics CAIM 2018, September 20-23, 2018, Chişinău.
- International conference Mathematics & Information technologies: research and education, MITRE-2019, Moldova, State University, June 24, 2019, Chișinău.
- LOOPS 2019 Conference, Budapest University of Technology and Economics, July 7-July 11, 2019, Hungary.
- The 5th International Conference of Mathematical Society of the Republic of Moldova, dedicated to the 55th anniversary of the foundation of Vladimir Andrunachievici Institute of Mathematics and Computer Science (IMCS-55), September 30, 2019, Chișinău.

Thesis publications. A total of 16 scientific papers have been published, comprising 6 articles in peer-reviewed journals (1 article without co-authors) and 10 abstracts at scientific conferences (7 abstracts without co-authors).

Thesis structure and thesis volume. The thesis is written in Romanian and contains an introduction, four chapters, general conclusions and recommendations, 109 bibliographic titles, 104 pages (including 91 pages of basic text).

keywords: quasigroup, loop, group, groupoid, isotope, automorphism, left identity element, right identity element, pseudo-automorphism, left Bol quasigroup (right), Moufang quasigroup, *WA*-quasigroup, *CI*-quasigroup, *i*-quasigroup, medial quasigroup, Neumann quasigroup, left-transitive quasigroups, *G*-property.

2. CONTENT OF THE THESIS

The structure of the thesis is represented by four chapters, which contain the theoretical results obtained in the research of the properties of non-associative algebraic systems with Moufang type identities.

The introduction formulates the topicality and importance of the research topic. In addition, the objectives, scientific novelty, and originality are mentioned. The scientific problem studied is presented with emphasis on the importance of the theoretical and applied value of the paper. A short analysis of the issues and publications on the topic of the thesis is presented. This section concludes with a summary of the content of the paper.

The first chapter – Analysis of the situation in the field of non-associative algebraic systems with Moufang type conditions - consisting of six paragraphs, has an introductory character. In this chapter, an analysis of the fundamental algebraic concepts is made. It contains the basic notions and theorems needed to present the thesis.

We often research quasigroups and loops in terms of their proximity to groups. The reversibility property also plays a big role in the group. In the **second Chapter - About some classes of quasigroups with invertibility properties (*WA*-, *OWIP*-, *CI*- quasigroups) -** two known classes of quasigroups that have an inverse property are investigated; a new class of quasigroups is introduced, a generalization of *WIP*-quasigroups (*OWIP*-quasigroups). The chapter consists of seven paragraphs in which objectives 1 and 3 are achieved.

The first three sections are dedicated to the results obtained in the *WA*-quasigroup research. Quasigroup $(Q,\cdot)$ with identities $xx \cdot yz = xy \cdot xz$ and $xy \cdot zz = xz \cdot yz$ is called a *WA-quasigroup* or *semimedial quasigroup* (in short: SM-quasigroup).

WA-quasigroups were investigated by T. Kepka in the paper “A note on *WA*-quasigroups” (1978). In this paper, T. Kepka also characterized simple WA-quasigroups. Applying an important result obtained by T. Kepka - any loop, isotopic to *WA*-quasigroup, is the Moufang commutative loop [23] - the following have been demonstrated:

Lemma 2.1.2. Any WA-quasigroup with a left identity element is a left Bol quasigroup.

Lemma 2.1.3. Any WA-quasigroup with a right identity element is a right Bol quasigroup.

Lemma 2.1.4. Any WA-quasigroup $(Q,\cdot)$ with a left (right) inverse property is a left (right) Bol quasigroup.

From Lemma 2.1.4. and the fact that quasigroup Bol on the left and Bol on the right is Moufang quasigroup Corollary 2.1.2. results.

Corollary 2.1.2. Any WA-quasigroup, which is an IP-quasigroup, is a Moufang quasigroup.

Isotope of the form $x \circ y = L_a^{-1}(L_a x \cdot y)$ $((x \circ y = R_a^{-1}(x \cdot R_a y))$ is called a *right (left) derivative operation* for $(Q,\cdot)$, generated by element a. By researching the derivative operations for the WA-quasigroup $(Q,\cdot)$, we obtained Theorem 2.1.2.

Theorem 2.1.2. Let $(Q,\cdot)$ be a WA-quasigroup. Then:

(i) the right derivative operation $(Q,\cdot)$ is a left Bol quasigroup,

(ii) the left derivative operation $(Q,\cdot)$ is a right Bol quasigroup.

Elements of the group $I_h(Q,\cdot) = \{\alpha \in M(Q,\cdot) |\ \alpha h = h\}$, where $M(Q,\cdot)$ is the group generated by all left and right translations of a quasigroup $(Q,\cdot)$, are called *inner mappings of* $(Q,\cdot)$ relative to the element $h \in Q$. Belousov proved (cf. [8]) that the group $I_h(Q,\cdot)$ is generated by all permutations of the form:

$$L_{x,y} = L_{x\circ y}^{-1} L_x L_y, \text{ where } (x \circ y)h = x \cdot yh,$$

$$R_{x,y} = R_{x\bullet y}^{-1} R_y R_x, \text{ where } h(x \bullet y) = hx \cdot y,$$

$$T_x = L_{\sigma x}^{-1} R_x, \text{ where } \sigma = R_h^{-1} L_h.$$

Paragraph 2.2. of the thesis is dedicated to WA-quasigroup automorphisms, inner permutations as automorphisms in relation to the left unit and in relation to arbitrary element, determining the necessary and sufficient conditions:

Theorem 2.2.1. In a WA-quasigroup $(Q,\cdot)$ with the left identity element f, inner permutations $L_{x,y}, R_{x,y}$ and T_x relative to $a \in Q$, are automorphisms in $(Q,\cdot)$ if and only if $a \in N_l$ and the following identity $xy \cdot a = xf \cdot ya$ is satisfied.

In paragraph 2.4. of this chapter, the properties of a new class of quasigroups, namely *OWIP*-quasigroups, were introduced by the student of V.D. Belousov, I.A. Florja, and the author of the thesis.

The quasigroup $(Q,\cdot)$ has a weak reversible property ($(Q,\cdot)$ is a *WIP*-quasigroup), if $x \cdot I(y \cdot x) = Iy$ for all $x, y \in Q$ and some permutation of the set Q. Loops with weakened inverse property loops have been researched by Osborn [24]. By introducing an α permutation in the given identity, a new class of quasigroups was obtained, namely *OWIP-quasigroups.*

Definition 2.4.1. Quasigroup $(Q,\cdot)$ is called a *$OWIP$-quasigroup*, if in $(Q,\cdot)$ the following identity is true

$$x \cdot I(y \cdot \alpha x) = Iy, \tag{2.24}$$

for all $x, y \in Q$, where I and α are some permutations of the set Q.

From identity (2.24), by applying the properties of permutations, a new identity (2.25) was obtained for this quasigroup class (2.25):

Lemma 2.4.1. In a $OWIP$-quasigroup $(Q,\cdot)$, the following identity holds:

$$I^{-1}(xz) \cdot \alpha x = I^{-1} z, \tag{2.25}$$

for any $x, z \in Q$.

By applying (2.25), the necessary and sufficient condition was found when the OWIP-quasigroup $(Q,\cdot)$ is an isotope of the LIP-loop $(Q,\circ)$. See Theorem 2.4.1.

Theorem 2.4.1. $OWIP$-quasigroup $(Q,\cdot)$ is an isotope of the LIP-loop $(Q,\circ)$, where

$$x \circ y = R_a^{-1} x \cdot L_b^{-1} y, xy = R_a x \circ L_b y \tag{2.26}$$

if and only if in $(Q,\cdot)$ the following equality is true

$$b \cdot I(I^{-1}(by) \cdot x) = R_{e_b}^{-1}\big(b \cdot I(I^{-1}b \cdot x)\big) \cdot y, \tag{2.27}$$

where $be_b = b,\ R_{e_b}v = ve_b$.

Assuming that any loop $(Q,\circ)$, isotopic to the $OWIP$-quasigroup $(Q,\cdot)$, is a LIP-loop, then in the quasigroup $(Q,\cdot)$, the following identity takes place:

$$z \cdot I(I^{-1}(zy) \cdot x) = R_{e_z}^{-1}\big(z \cdot I(I^{-1}z \cdot x)\big) \cdot y, \tag{2.28}$$

for any $x, y, z \in Q$, where I is a permutation of the set $Q, ze_z = z, R_{e_z}t = te_z$. It was obtained:

Theorem 2.4.2. Any loop $(Q,\circ)$, isotopic of the quasigroup $(Q,\cdot)$ with identity (2.28), is the left Bol loop.

The results on the right are presented analogously.

The last two sections of this chapter present the results obtained in the research of CI-quasigroups. CI-loops are classical objects of quasigroup theory. This loop class was defined by Rafael Artzy [11]. V. D. Belousov and B. V. Tsurkan in the joint work "Crossed invertible quasigroups (CI-quasigroups)", (1969) defined CI-quasigroups. Some applications of CI-quasigroups in cryptology are presented in the work by A. D. Keedwell (1999).

Definition 2.5.3. Groupoid $(Q,\cdot)$ with the identity

$$xy \cdot I_r x = y, \tag{2.30}$$

where I_r is a map of the set Q into itself, is called a left CI-groupoid.

Groupoid $(Q,\cdot)$ with the identity

$$I_l x \cdot yx = y, \tag{2.31}$$

where I_l is a map of the set Q into itself, is called a right CI-groupoid.

Groupoid $(Q,\cdot)$ with both identities (2.30) and (2.31) is called a CI-groupoid [13].

Loop $(Q,\cdot)$ satisfying one of the equivalent identities, $x \cdot yJx = y, xy \cdot Jx = y$, where J is a bijection of the set Q such that $x \cdot Jx = 1$, is called a *CI-loop*. Quasigroup $(Q,\cdot)$ with the identity $xy \cdot Jx = y$, where J is a map of the set Q, is called *CI-quasigroup*.

Notice, in this case, the mapping J is a permutation of the set Q [12]. In any CI-quasigroup, the permutation J is unique ([25], Lemma 2.25).

From the results of V. Izbash and N. Labo, it follows that the CI-groupoid on the left, in which the application I_r is bijective, is a CI-quasigroup. Any left CI-groupoid is a CI-quasigroup. Any CI-groupoid is a CI-quasigroup [13].

The case was investigated when a CI-quasigroup is an isotope of a group. The condition was found when the loop, isotopic to CI-quasigroup, is commutative. We see the Propositions:

Proposition 2.6.1. CI-quasigroup $(Q,\cdot)$ is the isotope of the group $(Q,\circ)$, with the isotopy (2.26) if and only if in $(Q,\cdot)$ the following equality is true:

$$\Big(x\big(y(zu)\big)\Big)\cdot v = y\cdot\big((xz\cdot v)\cdot u\big), \qquad (2.36)$$

for any $x, y, z, u, v \in Q$.

Proposition 2.6.3. If any loop $(Q,\circ)$, isotopic to CI-quasigroup $(Q,\cdot)$, is commutative, then the quasigroup $(Q,\cdot)$ is medial, and $(Q,\circ)$ is an abelian group.

In the **third Chapter – About a class of $\boldsymbol{i}$-quasigroups. Units on the right (left) in Bol-Moufang type quasigroups -** consisting of six sections, objectives 2 and 3 are achieved.

In this chapter, a new class of quasigroups called i-quasigroups is introduced by I. A. Florja and the author of the thesis.

Definition 3.1.2. Quasigroup $(Q,\cdot)$ is called an *i-quasigroup*, if in $(Q,\cdot)$ the following identity is true:

$$x(xy\cdot z) = y(zx\cdot x), \qquad (3.1)$$

where $x, y, z \in Q$.

The first example built by such a quasigroup is:

Example 3.1.1.

The set C of all complex numbers with the operation $x\circ y = ix - y$ is an *i*-quasigroup. Hence it was called *i*-quasigroup.

In the first paragraph of this chapter, the distributant of *i*-quasigroups was researched. By *distributant* of a quasigroup $(Q,\cdot)$ we mean the set D containing all elements $d \in Q$ such that $(x\cdot y)\cdot d = (x\cdot d)\cdot(y\cdot d), d\cdot(x\cdot y) = (d\cdot x)\cdot(d\cdot y)$ for all $x, y \in Q$. We know that not every quasigroup has an empty distributant. We highlight the following theorems:

Theorem 3.1.1. If i -quasigroup $(Q,\cdot)$ is an RIP-quasigroup, then $(Q,\cdot)$ is a Moufang quasigroup with a left identity element f and distributant $D = \{f\}$.

Theorem 3.1.4. i-quasigroup $(Q,\cdot)$ with nonempty distributant D is a left Bol quasigroup if and only if in $(Q,\cdot)$ the following equality is true:

$$xa\cdot xy = xx\cdot ay, \qquad (3.6)$$

for any $x, y \in Q$, where $a \in D$, a is a fixed element.

It has been shown that *i*-quasigroups with the right unity are Moufang loops, those with the left unity are the quasigroups with the left inverse property. The results obtained are presented in the following propositions:

Proposition 3.2.2. If i-quasigroup $(Q,\cdot)$ has right identity element e, then $(Q,\cdot)$ is a Moufang loop in wich $x^2y = yx^2$ for any $x, y \in Q$.

Proposition 3.2.3. Any i-quasigroup $(Q,\cdot)$ with left identity element f is a quasigroup with left inverse property and isotopic to LIP-loop $(Q,\circ)$, where $x \circ y = R_f^{-1}x \cdot y$.

As long as the i-quasigroup is unipotent, it was obtained that it is a Moufang quasigroup. We see the following proposition:

Proposition 3.2.4. If in i-quasigroup $(Q,\cdot)$ the equality $x^2 = f$ takes place for any $x \in Q$, where f is a fixed element, then $(Q,\cdot)$ is a Moufang quasigroup with left unit element f and an isotope of an abelian group.

In section 3.3. of this chapter, the right (left) alternative quasigroups were investigated, showing that they are Moufang loops. For i-quasigroups with elasticity it was obtained:

Proposition 3.3.3. In every i-quasigroup $(Q,\cdot)$ with identity of elasticity (3.21), a set of all local unit elements forms left Bol subquasigroup.

The relationship between some types of quasigroups that contain unity on the left with the Moufang quasigroups (in the sense of Belousov) is investigated.

Proposition 3.3.4. i-quasigroup $(Q,\cdot)$ with left identity element f is a Moufang quasigroup if and only if in $(Q,\cdot)$ the following identity is true:

$$zx \cdot x = zf \cdot xx, \tag{3.26}$$

for any $x, z \in Q$.

Certain pseudo-automorphisms of these quasigroups are characterized (in the sense of Pflugfelder).

Proposition 3.4.1. If permutation α of set Q of i-quasigroup $(Q,\cdot)$ with left identity element f is a pseudo-automorphism from the right of quasigroup $(Q,\cdot)$ with companion k, then k is a left Bol element.

Proposition 3.4.2. If in i-quasigroup $(Q,\cdot)$ with left unit f the translations L_a and R_b are right pseudo-automorphisms with companion k, then $a = e_k f, b = e_k$, where $ke_k = k$.

The last section of this chapter is dedicated to researching the existence of the right (left) unity in quasigroups with each of the 60 Bol-Moufang identities listed in the paper "Extra loops II. On loops with identities of Bol-Moufang type" by Fenyves. An identity based on a single binary operation is of Bol-Moufang type if «both sides consist of the same three different letters taken in the same order but one of them occurs twice on each side».

K. Kunen in "Quasigroups, loops and associative laws", (1996) studied the existence of unity in quasigroups with weak associativity and completely solved the problem of the existence of bilateral unity in quasigroups with Bol-Moufang identities, for each of the 60 identities. Kunen also shows which of the highlighted identities implies the laws: associative, flexibility, alternative to the right (left), thus solving the problem for associative quasigroups with each of the 60 Bol-Moufang identities and partly for some identities - the problem of unilateral unity existence. The issue of the existence of unilateral unity for Fenyves' identities remained open [26].

We use the list of 60 Bol-Moufang type identities given in [27]. Notice, there exist other definitions of Bol-Moufang type identities [28, 29].

We highlight some theorems:

Theorem 3.5.13. Quasigroup $(Q,\cdot)$ with identity F_{16} $(xy \cdot x)z = x(yx \cdot z)$ has a left unit and it has no right unit.

Theorem 3.5.20. Quasigroup $(Q,\cdot)$ with identity F_{35} $(yx \cdot x)z = (y \cdot xx)z$ has a right unit and it has no left unit.

Theorem 3.5.21. Quasigroup $(Q,\cdot)$ with identity F_{36} (RC identity) $(yx \cdot x)z = y(xx \cdot z)$ has a left unit and it has no right unit.

Theorem 3.5.32. Quasigroup $(Q,\cdot)$ with identity F_{49} $(xx \cdot y)z = x(xy \cdot z)$ has a left unit and it has no right unit.

This solves objective 2.

In **Chapter four – Left-transitive quasigroup. Neumann and Schweizer quasigroups –** the objectives are achieved, which refer to the research of the G-properties of the left-transitive quasigroups and Neumann and to the research of the relations of these quasigroups with the Moufang quasigroups, left (right) Bol, and other quasigroup classes.

In sections 4.1.-4.5., the results obtained in the research of the left-transitive quasigroups are formulated, namely: about the relationship of these quasigroups with other classes of quasigroups, about the nuclei, about morphisms, about G-properties.

Quasigroup $(Q,\cdot)$ is said to be *left-transitive* if in this quasigroup the identity $xy \cdot xz = yz$ holds. In some works, they are also called Ward quasigroups [17].

In the publication „On the foundations of quasigroups" by Stein (1957), it has been proven: if the quasigroup$(Q,\cdot)$ verifies the associativity $x \cdot yz = xy \cdot z$ (that is, this quasigroup is a group), then (23)- and (132)-parastrophes of this quasigroup verify the left transitive identity. He also proved that Ward quasigroups are isotopes of groups.

In section 4.2., the necessary and sufficient condition was found when the left-transitive quasigroup is the Moufang quasigroup, left F-quasigroup (Proposition 4.2.3., Theorem 4.2.2.).

Theorem 4.2.2. Any left-transitive quasigroup $(Q,\cdot)$ is a left F-quasigroup if and only if a translation R_f is an automorphism of the quasigroup $(Q,\cdot)$.

The left and right nuclei of the left-transitive quasigroup were characterized. It is well known that the sets N_l and N_r form subgroups of quasigroup. We have:

Theorem 4.3.1. Left nucleus $(N_l,\cdot)$ of left-transitive quasigroup $(Q,\cdot)$ is a normal subgroup of quasigroup $(Q,\cdot)$ which consists of elements of order two that lie in the center of a group $(Q,+)$.

Theorem 4.3.2. If left-transitive quasigroup $(Q,\cdot)$ has a non-empty right nucleus, then it is a commutative 2-group.

We know that the quasigroup $(Q,\cdot)$ is a G-quasigroup on the right, if any its element is the companion of the right pseudo-automorphism. The G-quasigroup on the left is defined analogously. And the quasigroup that is at the same time G-quasigroup on the right and G-quasigroup on the left is a G-quasigroup. G-quasigroups always have unity, i.e., the G-quasigroup is a loop. The importance of researching pseudo-automorphisms follows from V. D. Belousov's theorem: the loop $(L,\cdot)$ is a G-loop if and only if any element $x \in L$ is the companion of each pseudo-automorphism to the right and to each one to the left of a loop $(L,\cdot)$. V. D. Belousov's results pave the way for research into G-properties, i.e., the research into G-loops and G-quasigroups. There is a connection between the nuclei of quasigroups and the pseudo-automorphisms of quasigroups.

In section 4.5., the G-properties for the left-transitive quasigroup were investigated. The necessary and sufficient condition is formulated for it to be the right GA-quasigroup, namely, the group ${}_2\Pi_r^A$ (or the group ${}_3\Pi_r^A$) is transitive on the set Q.

Theorem 4.5.3. A left-transitive quasigroup $(Q,\cdot)$ is a right GA-quasigroup if and only if $(Q,\cdot)$ is an abelian 2-group.

In sections 4.6.-4.8., the results obtained for the Neumann quasigroups are formulated.

Quasigroup $(Q,\cdot)$ is said to be *Neumann quasigroup* if in this quasigroup the identity $(Q,\cdot)$ $x \cdot (yz \cdot yx) = z$ holds. Neumann's identity was obtained from the (13)-parastrophe of the quasigroup verifying the identity $xy \cdot z = y \cdot zx$. Neumann quasigroups are isotopes of abelian groups. We see from:

Theorem 4.6.3. Any Neumann quasigroup $(Q,\cdot)$ is an isotope of an abelian group $(Q,+)$ of the form $x \cdot y = x - y$.

From the research of autotopies and quasiautomorphisms, their general forms were obtained.

Corollary 4.6.2. Any autotopy of Neumann quasigroup $(Q,\cdot)$ has the form:

$$(L_a^{\cdot}, L_{Ib}^{\cdot}, L_{a\cdot Ib}^{\cdot})\theta_1, \tag{4.23}$$

where $L_a^{\cdot}, L_{Ib}^{\cdot}, L_{a\cdot Ib}^{\cdot}$ are translations of quasigroup $(Q,\cdot)$, $\theta_1 = I\theta \in Aut(Q,\cdot)$.

Proposition 4.7.2. In Neumann quasigroup $(Q,\cdot)$, any quasiautomorphism γ has the form

$$\gamma = R_a\gamma' = L_b\gamma'', \tag{4.29}$$

where γ', γ'' are quasigroup $(Q,\cdot)$ automorphisms, b and a - any elements of Q and vice versa, the γ permutation in (4.29) is a quasiautomorphism.

Applying Theorem 4.6.3., some properties of the Neumann quasigroups have been easily demonstrated.

Corollary 4.6.3.

1. Any Neumann quasigroup $(Q,\cdot)$ is unipotent and has a right unit element;
2. Any loop which is an isotope of Neumann quasigroup is a commutative group;
3. Any Neumann quasigroup is a medial quasigroup;
4. Any Neumann quasigroup is a left Bol quasigroup;
5. Any Neumann quasigroup is a Moufang quasigroup;
6. Core of Neumann quasigroup is a distributive groupoid;
7. The right nucleus of Neumann quasigroup $(Q,\cdot)$ consists of elements of the set Q, such that $a = -a$.

Quasigroup $(Q,\cdot)$ with identity $yz \cdot yx = xz$ is called a *Schweizer quasigroup*. The notion of Neumann quasigroup has been shown to coincide with that of Schweizer quasigroup.

The following takes place:

Theorem 4.6.5. Any Schweizer quasigroup $(Q,\cdot)$ is a Neumann quasigroup and vice versa.

Research on the *G*-properties of these quasigroups found that any Neumann quasigroup $(Q,\cdot)$ is a GA-quasigroup, that is, groups ${}_3\Pi_r^A$ and ${}_3\Pi_l^A$ act on the set Q transitively.

Theorem 4.8.1. Any Neumann quasigroup $(Q,\cdot)$ is a GA-quasigroup.

Acknowledgments. I express my sincere acknowledgments to the scientific leader *Victor Alexeevich Shcerbacov* for determining the field of research, for formulating research objectives, for the knowledge I gained during the four years of my doctorate, for the help he gave me in making the publications and forming the thesis.

With special consideration and gratitude, I bring my acknowledgments to the professor, the candidate in physical and mathematical sciences *Ivan Arhipovici Florea*, who guided me through the first steps in the theory of quasigroups, the first research of different classes of quasigroups. His publications serve me as benchmarks of the following research.

I express my gratitude to the director of the doctoral school of Mathematics and Information Science, academician $\boxed{Mitrofan\ Mihailovici\ Cioban}$, for his patience, guidance, and encouragement.

Special thanks go to the members of the steering committee, Dr. hab., Corr. Mem. *Cojocaru Svetlana*, Acad. *Arnautov Vladimir*, and Dr. hab. *Chiriac Liubomir* for their valuable pieces of advice and recommendations.

Special thanks to Dr. hab., Corr. Mem. *Gaindric Constantin* and Dr., associate professor *Sârbu Parascovia* for the competent consultations and advice offered.

I dedicate this work to *my late parents, Nicolae* and *Olga*, with love and gratitude.

3. GENERAL CONCLUSIONS AND RECOMMENDATIONS

Research conducted in the doctoral thesis "**Morphisms and properties of non-associative algebraic systems with Moufang conditions**" fully corresponds to the purpose and objectives set out in the introduction.

The quasigroup class contains that of the groups (any group is a quasigroup), a field that experienced an extraordinary development in the twentieth century and continues to develop rapidly today. The notion of quasigroup is more general than that of a group, so it is found as an algebraic equivalent in a wider range of environmental problems in which we exist and requires specific approaches, which are missing in group theory.

The important scientific problem solved is the application of morphism relations to the research of properties and units of non-associative algebraic systems with Moufang-type conditions leading to the description of important new relations between quasigroup classes.

The main results of the paper are new. The analysis of the obtained results allows us to highlight the following general results:

1. It has been established that any WA-quasigroup, which is IP-quasigroup, is a Moufang quasigroup;

   It turned out that in the WA-quasigroup with the left unit the internal permutations in relation to the unit are automorphisms of the quasigroup; for internal permutations with respect to the element $a \in Q$, the necessary and sufficient condition was found when there are automorphisms;
2. By researching the generalized WIP-quasigroups it was found the condition when this quasigroup is an isotope of the loop with the left inverse property. Assuming that any isotope loop of $OWIP$-quasigroup is a LIP-loop, we obtained a new identity in this quasigroup, for which it was found the relationship with the left Bol loop;
3. Defining a new class of quasigroups (i-quasigroups), their relationships with other classes of quasigroups were researched. Assuming that the i-quasigroup $(Q,\cdot)$ is idempotent, it turned out to be the left Bol quasigroup, right Stein quasigroup. The relationship be-

tween some types of quasigroups containing left unit with Moufang quasigroups has been described;

4. The problem of the existence of the unilateral unit in quasigroups with Bol-Moufang type identities, listed in the paper "Extra loops II. On loops with identities of Bol-Moufang type" by F. Fenyves (1969) has been solved;
5. It has been shown that the notion of Neumann quasigroup coincides with that of Schweizer quasigroup.

The thesis proposed for defense contains the complete solution of the problem in the application of morphism relations to the research of properties and units of non-associative algebraic systems with Bol-Moufang type conditions leading to the description of important new relations between quasigroup classes, contains the complete solution of the problem of the existence of the unilateral unit in quasigroups with Bol-Moufang type identities.

Recommendation:

1. The solution of the problem of the existence of the unilateral unit for quasigroups satisfying Bol-Moufang type identities can be used in the research of quasigroups with Bol-Moufang type identities.
2. Two new classes of quasigroups are defined in the thesis: i-quasigroups and $OWIP$-quasigroups. The paper investigates some properties of these classes of quasigroups, but their general theory is to be developed.
3. It is recommended that the results obtained be applied to the development of optional courses for master and doctoral students.

REFERENCES

[1] MOUFANG, R., Zur Structur von Alternativ Korpern, *Math. Ann.,* vol. 110, pp. 416-430, 1935. ISSN: 0025-5831.

[2] ALBERT, A. A., Quasigroups. I, *Transactions of the American Mathematical Society,* vol. 54, no. 1, pp. 507-519, 1943. ISSN: 0002-9947

[3] ALBERT, A. A., Quaisgroups. II, *Trans. Amer. Math. Soc.,* vol. 55, pp. 401-419, 1944. ISSN: 0002-9947

[4] TOYODA, K., On axioms of linear functions, *Proc. Imp. Acad. Tokyo,* vol. 17, pp. 221-227, 1941. ISSN: 03609-9846

[5] BRUCK, R. H., Some results in the theory of quasigroups, *Trans. Amer. Math. Soc.,* vol. 55, pp. 19-52, 1944. ISSN: 0002-9947

[6] BAER, R., Nets and groups. I, *Trans. Amer. Math. Soc.,* vol. 46, pp. 110-141, 1939. ISSN: 0002-9947

[7] BAER, R., Nets and groups. II, *Trans. Amer. Math. Soc.,* vol. 47, no. 2, pp. 435-439, 1940. ISSN: 0002-9947

[8] БЕЛОУСОВ, В. Д., Основы теории квазигрупп и луп, Москва: Наука, 1967.

[9] ФЛОРЯ, И. А., Квазигруппы Бола, *Исследования по общей алгебре, АН МССР,* 1965.

[10] ФЛОРЯ, И. А., Связь левотранзитивных квазигрупп с квазигруппами Бола, *Сети и квазигруппы,* pp. 203-216, 1976.

[11] ARTZY, R., On loops with a special property, *Proc. Amer. Math. Soc.,* no. 6, pp. 448-453, 1955. ISSN: 0002-9939

[12] БЕЛОУСОВ, В. Д. и ЦУРКАН, Б. В., Скрещенно-обратимые квазигруппы (CI-квазигруппы), *Изв. Выш. Учебн. Завед. Математика.,* т. 82(3), pp. 21-27, 1969. 0021-3446 (print)

[13] IZBASH, V. and LABO, N., Crossed-inverse-property groupoids, *Buletinul Academiei de Științe a Republicii Moldova. Matematica,* no. 2(54), pp. 101-106, 2007. ISSN: 1024-7696

[14] URSU, V. I., On the pre-ordering of automorphic loops and Moufang loops, *Rev. Roumaine Math. Pures Appl.,* vol. 64, no. 1, pp. 49-55, 2019. ISSN: 0035-3965

[15] SYRBU, P. N., On loops with universal elasticity, *Quasigroups Related Systems,* no. 3, pp. 41-54, 1996. ISSN: (Print) 1561-2848

[16] DUDEK, W. A. and MONZO, R. A., The structure of idempotent translatable quasigroups, *Bull. Malays. Math. Sci. Soc.,* vol. 43, no. 2, pp. 1603-1621, 2020. ISSN: 0126-6705

[17] CHATTERJEA, S. K., On Ward quasigroups, *Pure Math. Manuscript*, no. 6, pp. 31-34, 1987.

[18] DENES, J. and KEEDWELL, A. D., Some applications of non-associative algebraic systems in cryptology, *P.U.M.A.*, vol. 12(2), pp. 147-195, 2002. ISSN: 1218-4586

[19] HOROSH, G., SHCHERBACOV, V., TCACHENCO, A. and YATSKO, T., On some groupoids with Bol-Moufang type identities, 2019. [Online]. Available: arxiv.org.1904.v1.

[20] IZBASH, V. I. and SHCHERBACOV, V., On quasigroups with Moufang identity, in *In Abstracts of The Third International Conference in memory of M.I. Kargapolov (1928-1976)*, Krasnoyarsk, Russian Federation, 1993.

[21] EVANS, T., Homomorphisms of non-associative systems, *J. London Math. Soc.*, vol. 24, pp. 254-260, 1949. ISSN: 0024-6107

[22] PHILLIPS, J. D. and VOJTECHOVSKY, P., The varieties of loops of Bol-Moufang type, *Algebra Universalis*, vol. 54(3), pp. 259-271, 2005. ISSN: 0002-5240

[23] KEPKA, T., A note on WA-quasigroups, *Acta Univ. Carolin. Math. Phis.*, vol. 19(2), pp. 61-62, 1978. ISSN: 0001-7140

[24] OSBORN, M., Loops with the weak inverse property, *Pacif. J. Math.*, vol. 10, no. 1, pp. 295-304, 1960. ISSN: 0030-8730

[25] SHCHERBACOV, V. A., Elements of Quasigroup Theory and Applications, Boca Raton: CRC Press, 2017. ISBN: 9781315120058

[26] KUNEN, K., Quasigroups, loops and associative laws, *J. Algebra*, vol. 185(1), pp. 194-204, 1996. ISSN: (Print) 0021-8693

[27] JAYEOLA, T. G., ILOJIDE, E., OLATINWO, M. O. and SMARANDACHE, F., On the Classification of BolMoufang Type of Some Varieties of Quasi Neutrosophic Triplet Loop (Fenyves BCI-Algebras), in *Symmetry*, 2018. ISSN: 2073-8994

[28] AKHTAR, R., ARP, A., KAMINSKI, M., VAN EXEL, J., VERNON, D. and WASHINGTON, C., The varieties of Bol-Moufang quasigroups defined by a single operation, *Quasigroups Related Systems*, vol. 20(1), pp. 1-10, 2012. ISSN: (Print) 1561-2848

[29] COTE, B., HARVILL, B., HUHN, M. and KIRCHMAN, A., Classification of loops of generalized Bol-Moufang type, *Quasigroups Related Systems*, vol. 19(2), pp. 193-206, 2011. ISSN: (Print) 1561-2848

LIST OF THE AUTHOR'S PUBLICATIONS ON THE THESIS

Articles in journals rated in SCOPUS:

1. DIDURIK, N. N. and FLORJA, I. A., A note on left loops with WA-property, *Quasigroups and Related Systems*, vol. 24, no. 2, pp. 186-196, 2016. ISSN (Print) 1561-2848
2. DIDURIK, N. N. and SHCHERBACOV, V. A., On definition of CI-quasigroup, *Romai Journal*, vol. 13, no. 2, pp. 55-58, 2017. ISSN (Print) 1841-5512
3. DIDURIK, N., Some properties of left-transitive quasigroups, *Buletinul Academiei de Ştiinţe a Republicii Moldova,* vol. 87, no. 2, pp. 85-94, 2018. ISSN: 1024-7696
4. DIDURIK, N. N. and FLORJA, I. A., Some properties of i-quasigroups, *Quasigroups and Related Systems*, vol. 28, no. 2, pp. 183-194, 2020. ISSN (Print) 1561-2848
5. DIDURIK, N. and SHCHERBACOV, V., Units in quasigroups with classical Bol-Moufang type identities, *Comment. Math. Univ. Carolin.*, vol. 61, no. 4, pp. 427-435, 2020. ISSN 0010-2628

Articles arXiv preprint:

6. DIDURIK, N. and SHCHERBACOV, V., Some properties of Neumann quasigroups, 2018. [Online]. Available: https://arxiv.org/abs/1809.07095v1.

Contributions (articles, abstracts, theses) to conference materials and other scientific papers:

7. DIDURIC, N., CI-quasigrupuri, în *"Tendinţe contemporane ale dezvoltării ştiinţei: viziuni ale tinerilor cercetători": Conferinţa Ştiinţifică a Doctoranzilor*, Chişinău, 2017, pp. 25-29. ISBN: 978-9975-108-66-9
8. DIDURIK, N., Generalized WIP-quasigroups, in *The Fourth Conference of Mathematical Society of the Republic of Moldova: dedicated to the centenary of Vladimir Andrunachievici (1917-1997): Proceedings CMSM 4*, Chişinău, 2017, pp. 67-70. ISBN: 978-9975-71-915-5
9. DIDURIK, N. N. and SHCHERBACOV, V. A., On definition of CI-quasigroups, in *The 25rd Conference on Applied and Industrial Mathematics CAIM*, Iaşi, 2017, p. 87. ISSN: 2038-0909
10. DIDURIK, N., On left-transitive quasigroups, in *The 25rd Conference on Applied and Industrial Mathematics CAIM*, Iaşi, 2017, p. 86. ISSN: 2038-0909

11. ДИДУРИК, Н., А-псевдоавтоморфизмы левых транзитивных квазигруп, in *International conference on mathematics, informatics and information technologies dedicated to the illustrious scientist Valentin Belousov*, Bălți, 2017, pp. 39-40. ISBN: 978-9975-3214-7-1
12. DIDURIK, N., Some properties of Neumann quasigroups, in *The 26th Conference on Applied and Industrial Mathematics CAIM 2018*, Chișinău, 2018, p. 93.
13. DIDURIC, N., Despre unele proprietăți ale quasigrupurilor cu identitatea lui Neumann, in *Tendințe contemporane ale dezvoltării științei: viziuni ale tinerilor cercetători. Conferința Științifică a Doctoranzilor*, Chișinău, 2018, pp. 11-14. ISBN: 978-9975-108-66-9
14. DIDURIK, N., i-Quasigroups, in *International conference Mathematics & Information technologies: research and education, MITRE-2019*, Chișinău, 2019, pp. 26-27.
15. DIDURIK, N. and SHCHERBACOV, V., Units in quasigroups with non-classical Bol-Moufang type identities, in *The 5th International Conference of Mathematical Society of the Republic of Moldova, dedicated to the 55th anniversary of the foundation of Vladimir Andrunachievici Institute of Mathematics and Computer Science (IMCS-55)*, Chișinău, 2019, pp. 57-60.
16. SHCHERBACOV, V. and DIDURIK, N., Units in quasigroups with Bol-Moufang type of identities, in *LOOPS Conference Budapest University of Technology and Economics*, Hungary, 2019, p. 49.

ADNOTARE

la teza de doctorat "**Morfismele şi proprietăţile sistemelor algebrice neasociative cu condiţii de tip Moufang**", prezentată de către Diduric Natalia pentru conferirea titlului științific de doctor în ştiinţe matematice, specialitatea 111.03 - Logică Matematică, Algebră şi Teoria Numerelor, Chişinău, 2022.

Structura tezei: teza este scrisă în limba română şi conţine introducere, patru capitole, concluzii generale şi recomandări, 109 titluri bibliografice, 104 pagini (inclusiv 91 pagini de text de bază). Rezultatele obţinute sunt publicate în 16 lucrări ştiinţifice.

Cuvinte-cheie: cvazigrup, buclă, izotop, pseudoautomorfism, cvazigrup Bol la stânga (la dreapta), cvazigrup Moufang, *WA*-cvazigrup, *i*-cvazigrup, *G*-proprietăţi.

Domeniul de studiu al tezei: algebră, în special, teoria cvazigrupurilor cu identităţi, inclusiv identităţile de tip Bol-Moufang, proprietăţile sistemelor algebrice neasociative.

Scopul şi obiectivele lucrării. Scopul lucrării este cercetarea proprietăţilor sistemelor algebrice neasociative cu identităţi de tip Bol-Moufang. Pentru atingerea acestui scop au fost definite următoarele obiective: cercetarea relațiilor *WA*-, *CI*-cvazigrupurilor, cvazigrupurilor tranzitive la stânga şi Neumann cu cvazigrupurile Moufang, Bol la stânga, Bol la dreapta ș.a.; cercetarea existenței unității unilaterale în cvazigrupuri cu identități de tip Bol-Moufang, enumerate în lucrarea lui F. Fenyves "Extra loops II. On loops with identities of Bol-Moufang type", (1969); cercetarea morfismelor, proprietăților, relațiilor cu alte clase de cvazigrupuri ale cvazigrupurilor noi definite în lucrare (*i*-cvazigrupuri și *OWIP*-cvazigrupuri); cercetarea *G*-proprietăților cvazigrupurilor tranzitive la stânga și Neumann.

Noutatea şi originalitatea ştiinţifică. Toate rezultatele prezentate în teză sunt noi şi originale. Au fost cercetate diverse clase de cvazigrupuri (*WA*-, *CI*-cvazigrupuri, cvazigrupuri tranzitive la stânga, Neumann ș.a.). Au fost introduse şi cercetate două clase noi de cvazigrupuri (*WIP*-cvazigrupuri generalizate, *i*-cvazigrupuri). Au fost cercetate clase de cvazigrupuri izotope grupurilor. Sunt descrise proprietăţile unor clase de cvazigrupuri inversabile. Au fost cercetate conexiuni între clasele de cvazigrupuri studiate și cvazigrupurile clasice Moufang, Bol ș.a. Sunt determinate formele generale ale automorfismelor, pseudoautomorfismelor și cvaziautomorfismelor acestor cvazigrupuri.

Problema ştiinţifică importantă soluţionată în domeniul respectiv constă în cercetarea diferitelor relații de tip morfisme (autotopii, pseudoautomorfisme, *G*-proprietăți) și noțiunilor (distributanților, nucleelor) în sistemele algebrice neasociative cu condiții de tip Bol-Moufang ce conduc la descrierea unor relații importante noi între clasele studiate de cvazigrupuri (inclusiv și clasele noi introduse).

Semnificația teoretică și valoarea aplicativă a lucrării este determinată de obţinerea unor rezultate noi în cercetarea sistemelor neasociative cu identități de tip Bol-Moufang. Lucrarea poartă un caracter teoretic.

Implementarea rezultatelor științifice. Rezultatele lucrării pot fi utilizate în predarea cursurilor de specialitate pentru studenții, masteranzii și doctoranzii de la specialitățile de matematică.

ANNOTATION

of the doctoral thesis “**Morphisms and properties of non-associative algebraic systems with Moufang type conditions**”, presented by Diduric Natalia for conferring the scientific title of Doctor in mathematical sciences, speciality 111.03 - Mathematical Logic, Algebra, and Number Theory, Chisinau, 2022.

Thesis structure: the thesis is written in Romanian and contains an introduction, four chapters, general conclusions and recommendations, 109 bibliographic titles, 104 pages (including 91 pages of basic text). The obtained results are published in 16 scientific papers.

Keywords: quasigroup, loop, isotope, pseudo-automorphism, left Bol (right) quasigroup, Moufang quasigroup, *WA*-quasigroup, *i*-quasigroup, *G*-properties.

Thesis field of study: algebra, especially, the theory of quasigroups with identities including Bol-Moufang-type identities, properties of non-associative algebraic systems.

The purpose and objectives of the paper. The aim of the paper is to investigate the properties of non-associative algebraic systems with Bol-Moufang type identities. To achieve this goal, the following objectives have been defined: research on the relations of *WA*-, *CI*-quasi-groups, transitive on the left and Neuman with the quasigroups Moufang, Bol on the left, on the right, etc.; research of the existence of unilateral unity in quasigroups with Bol-Moufang type identities, enumerated in the work of F. Fenyves “Extra loops II. On loops with identities of Bol-Moufang type”, (1969); research of morphisms, properties, relationships with other classes of quasigroups of newly defined quasigroups (*i*-quasigroups and *WIP*-generalized quasigroups); research on the *G*-properties of left transitive quasigroups and Neumann.

Scientific novelty and originality. All the results presented in the thesis are new and original. Diverse classes of quasigroups known earlier (*WA*-, *CI*-quasigroups, transitive left quasigroups, Neumann, etc.) were researched. Two new classes of quasigroups were introduced and researched (*i*-quasigroups, *WIP*-generalized quasigroups). Isotope group quasigroup classes were investigated. The properties of some classes of invertible quasigroups were described. Connections between the studied quasigroup classes and the classical quasigroups Moufang, Bol, etc. were investigated. The general forms of the automorphisms, pseudo-automorphisms, and quasiautomorphisms of these quasigroups were determined.

The important scientific problem solved consists in the research of different morphisms (autotopies, pseudoautomorphisms, *G*-properties) and notions (distributant, nucleus) in non-associative algebraic systems with Bol-Moufang conditions that lead to the description of important new relationships between the classes studied by quasigroups (including newly introduced classes).

The theoretical importance and applicative value of the thesis are determined by obtaining new results in the research of non-associative systems of the Bol-Moufang type. The paper is of theoretical character. The methods developed in the paper allowed solving the problems.

Implementation of scientific results. The results of the paper can be used in teaching specialized courses for students, masters and doctoral students in mathematics.

АННОТАЦИЯ

диссертации **«Морфизмы и свойства неассоциативных алгебраических систем с условиями типа Муфанг»**, представленной Дидурик Натальей для присвоения ученой степени доктора математических наук по специальности 111.03 – Математическая логика, Алгебра и Теория чисел, Кишинев, 2022 год.

Структура диссертации: диссертация написана на румынском языке и содержит введение, четыре главы, общие выводы и рекомендации, список литературы из 109 публикаций, 104 страницы (в том числе 91 страница основного текста). Результаты опубликованы в 16 научных работ.

Ключевые слова: квазигруппа, лупа, изотоп, псевдоавтоморфизм, левая (правая) квазигруппа Бола, квазигруппа Муфанг, WA-квазигруппа, i-квазигруппа, G-свойства.

Область исследования: алгебра, в частности, теория квазигрупп с тождествами, в том числе тождества типа Бола-Муфанг, свойства неассоциативных алгебраических систем.

Цель и задачи исследования. Целью диссертации является исследование свойств неассоциативных алгебраических систем с тождествами типа Бола-Муфанг. Для достижения этой цели были определены следующие задачи: исследование связи *WA*-, *CI*-квазигрупп, левотранзитивных и Неймана с квазигруппами Муфанг, левой и правой Бола и др.; исследование существования односторонней единицы в квазигруппах с тождествами типа Бола-Муфанг, перечисленные в работе Ф. Фенивса, “Extra loops II. On loops with identities of Bol-Moufang type”, (1969); исследование морфизмов, свойств и взаимосвязей с другими классами квазигрупп новых квазигрупп, определенных в работе (i-квазигруппы и обобщенные *WIP*-квазигруппы); исследование *G*-свойств левотранзитивных квазигрупп и квазигрупп Неймана.

Научная новизна и оригинальность. Все результаты, представленные в диссертации, являются новыми и оригинальными. Были исследованы различные классы квазигрупп (*WA*-, *CI*-квазигруппы, левотранзитивные квазигруппы, Неймана и др.). Были введены и исследованы два новых класса квазигрупп (обобщенные *WIP*-квазигруппы, *i*-квазигруппы). Исследованы классы квазигрупп изотопных группам. Описываются свойства некоторых классов обратимых квазигрупп. Исследованы связи между изучаемыми классами квазигрупп и классическими квазигруппами Муфанг, Бол и др. Определены общие формы автоморфизмов, псевдоавтоморфизмов и квазиавтоморфизмов этих квазигрупп.

Решенная важная научная проблема состоит в исследовании различных морфизмов (автотопий, псевдоавтоморфизмов, G-свойств) и понятий (дистрибутант, ядро) в неассоциативных алгебраических системах с условиями Бола-Муфанг, которые приводят к описанию новых важных взаимосвязей между изученными классами квазигрупп (включая и новые введенные классы).

Теоретическая значимость и прикладное значение работы определяется получением новых результатов в исследовании неассоциативных систем с тождествами типа Бола-Муфанг. Работа носит теоретический характер.

Внедрение научных результатов. Результаты работы могут быть использованы при преподавании специализированных курсов для студентов, магистров и аспирантов математических специальностей.

Didurik, Natalia

MORPHISMS AND PROPERTIES OF NON-ASSOCIATIVE ALGEBRAIC SYSTEMS WITH MOUFANG TYPE CONDITIONS

SPECIALITY 111.03 - MATHEMATICAL LOGIC, ALGEBRA, AND NUMBER THEORY

PhD in Mathematics Sciences thesis summary

Approved for printing: 04.02.2022 Paper size 60x84 1/16

Offset paper. Type offset. Drawing 15 ex.

Print sheets: 1,6

OOO „IMPRESO” Sverdlova street, 92, Tiraspol, MD-3300